B.Tech. Project:
Spatially Coupled LDPC Codes Over Fading Channels

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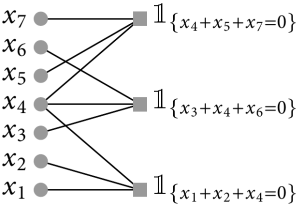
Outline

Codes and Parity-check Matrices

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \quad (1)$$



Tanner graph for **H** (Eqn. 1)*

Codeword $\mathbf{x} \in$ linear code $C \iff \mathbf{H}\mathbf{x}^{\mathsf{T}} = \mathbf{0}$

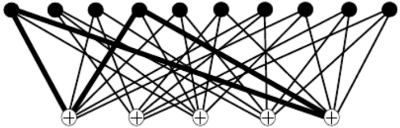
*Tom Richardson and Ruediger Urbanke. *Modern Coding Theory*. USA: Cambridge University Press, 2008. ISBN: 0521852293

LDPC Codes and Protographs

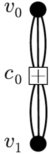
Low-density parity-check codes

$$\mathbf{H} = \begin{bmatrix} 1&0&1&1&1&0&1&1&0&0 \\ 0&1&1&0&0&1&1&0&1&1 \\ 1&1&0&1&1&1&0&1&0&0 \\ 0&1&1&0&1&0&1&0&1&1 \\ 1&0&0&1&0&1&0&1&1&1 \end{bmatrix} \quad (2)$$

$$\mathbf{B} = \begin{bmatrix} 3 & 3 \end{bmatrix} \quad (3)$$



(a) (3,6)-regular Tanner graph (Eqn. 2)*



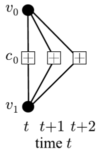
(b) (3,6)-regular protograph (Eqn. 3)†

*Costello J., Lara Dolecek, T.E. Fuja, et al. "Spatially Coupled Sparse Codes on Graphs - Theory and Practice". In: *Communications Magazine, IEEE* (Oct. 2013)

†David G. M. Mitchell, Michael Lentmaier, and Daniel J. Costello. "Spatially Coupled LDPC Codes Constructed From Protographs". In: *IEEE Transactions on Information Theory* 9 (2015)

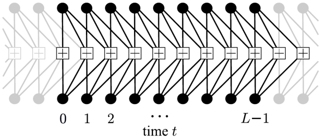
Spatial Coupling

$$\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{B}_0 \\ \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix}_{(w+1)b_c \times b_v} \tag{4}$$



(a) edge-spread (3,6)-regular protograph (Eqn. 4)*

$$\mathbf{B}_{[0,L-1]} = \begin{bmatrix} \mathbf{B}_0 & & \\ \vdots & \ddots & \\ \mathbf{B}_w & & \mathbf{B}_0 \\ & \ddots & \vdots \\ & & \mathbf{B}_w \end{bmatrix}_{(L+w)b_c \times Lb_v} \tag{5}$$



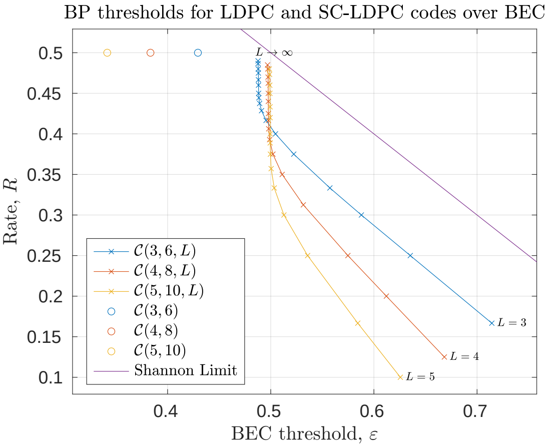
(b) $\mathcal{C}(3,6,L)$ ensemble protograph (Eqn. 5)*

*David G. M. Mitchell, Michael Lentmaier, and Daniel J. Costello. "Spatially Coupled LDPC Codes Constructed From Protographs". In: *IEEE Transactions on Information Theory* 9 (2015)

Performance of SC-LDPC Codes

$$\lim_{w,L\to\infty} R(J,K,L,w) = 1 - \frac{J}{K}$$

$$\lim_{w,L\to\infty} \mathtt{h}^{\mathrm{BP}}(J,K,L,w) = \mathtt{h}^{\mathrm{MAP}}(J,K)$$



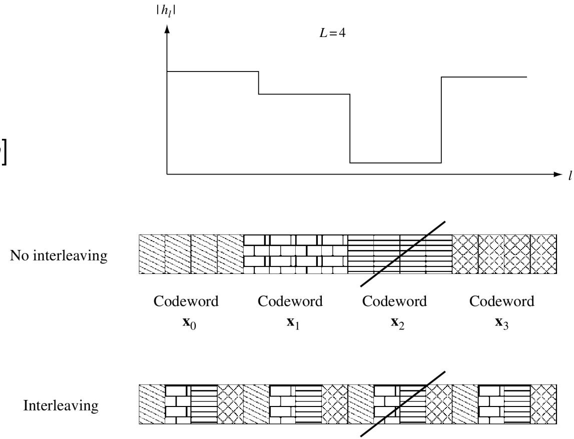
Channel Model and Interleaving

$$y[m] = \sum_l h_l[m] x[m-l] + w[m]$$

where $h_l[m] \sim \mathcal{CN}(0, \sigma_l^2)$

Single-tap model:

$$y[m] = h[m]x[m] + w[m]$$

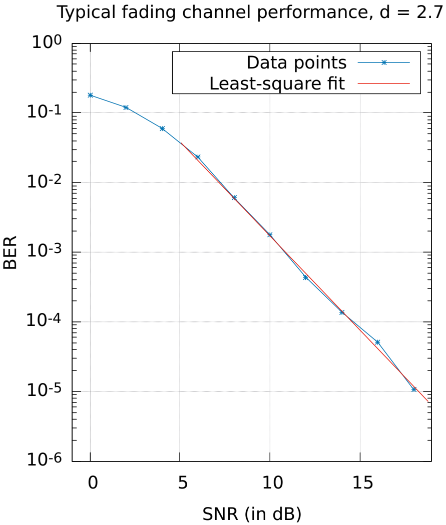


(David Tse and Pramod Viswanath. *Fundamentals of Wireless Communication*. USA: Cambridge University Press, 2005. ISBN: 0521845270)

Diversity

$$d = -\lim_{\mathrm{SNR}\to\infty} \frac{\log P_e}{\log \mathrm{SNR}}$$

calculated as the slope of a *least-square fit* line



Comparison With Prior Work

Work	Fading	Interleaving
Najeeb ul Hassan, Michael Lentmaier, Iryna Andriyanova, et al. "Improving code diversity on block-fading channels by spatial coupling". In: 2014	block	no
Yunlong Zhao, Yi Fang, and Zhaojie Yang. "Interleaver Design for Small-Coupling-Length Spatially Coupled Protograph LDPC-Coded BICM Systems Over Wireless Fading Channels". In: *IEEE Access* (2020), Sebastian Cammerer, Xiaojie Wang, Yingyan Ma, et al. "Spatially Coupled LDPC Codes and the Multiple Access Channel". In: 2019	IID	yes (BICM)
this work	correlated	yes (block)

Simulation of Fading Channels

Input: $\mathcal{C}(J, K, L)$ ensemble; SNR, f_d, $\sigma^2(=1)$ of fading channel
Output: BER

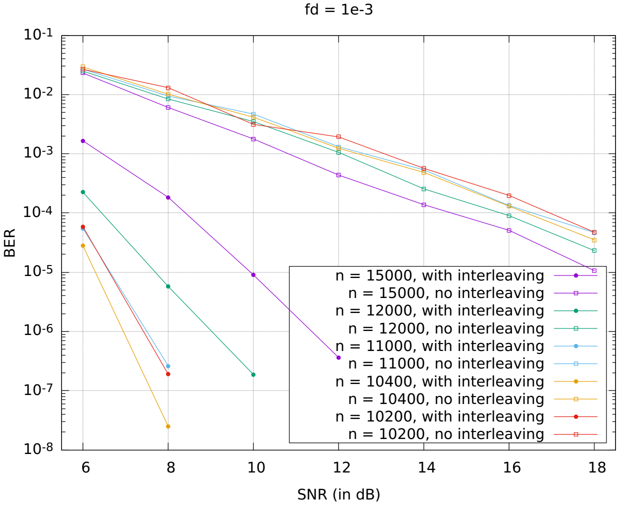
```
total errors ← 0, total bits ← 0
generate a C(J,K,L) code of blocklength n
repeat                                        ▷ averaging over time
    encode, interleave, BPSK modulate
    generate channel realization
    send through channel
    demodulate, deinterleave, decode
    BER ← total errors / total bits
until BER converges
```

BP implementation, Rayleigh process generated using IT++, http://itpp.sourceforge.net/4.3.1/

Fast Fading Channel

$(F_d T_s =)\ f_d = 10^{-3}$:

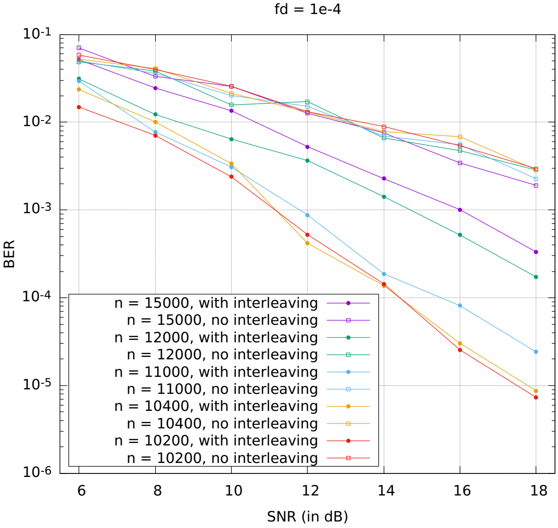
n	d^*	Diversity	
		direct	interleaved
15000	3	2.73	6.14
12000	6	2.53	7.69
11000	11	2.31	11.6
10400	26	2.41	15.2
10200	51	2.26	12.5



Moderately Fast Fading Channel

$f_d = 10^{-4}$:

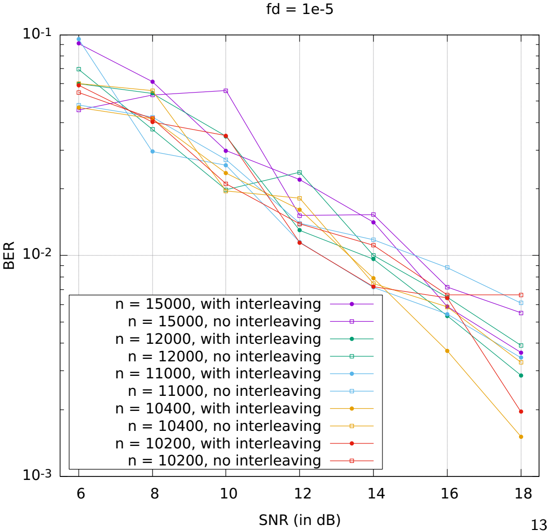
n	d^*	Diversity	
		direct	interleaved
15000	3	1.29	1.81
12000	6	1.05	1.82
11000	11	1.09	2.58
10400	26	1.03	2.99
10200	51	1.08	2.86



Slow Fading Channel

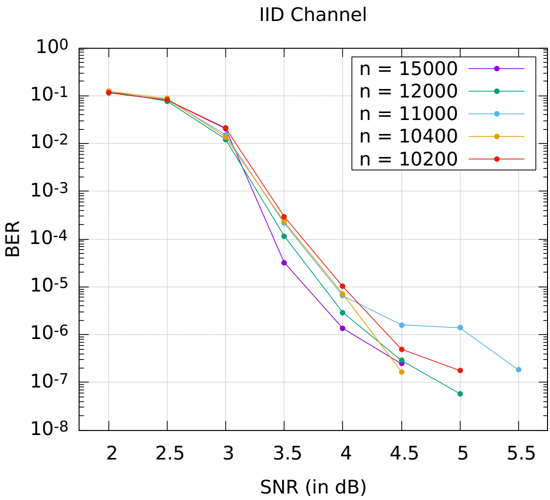
$f_d = 10^{-5}$:

n	d^*	Diversity	
		direct	interleaved
15000	3	0.9	1.17
12000	6	0.99	1.17
11000	11	0.79	1.13
10400	26	1.1	1.26
10200	51	0.83	1.2

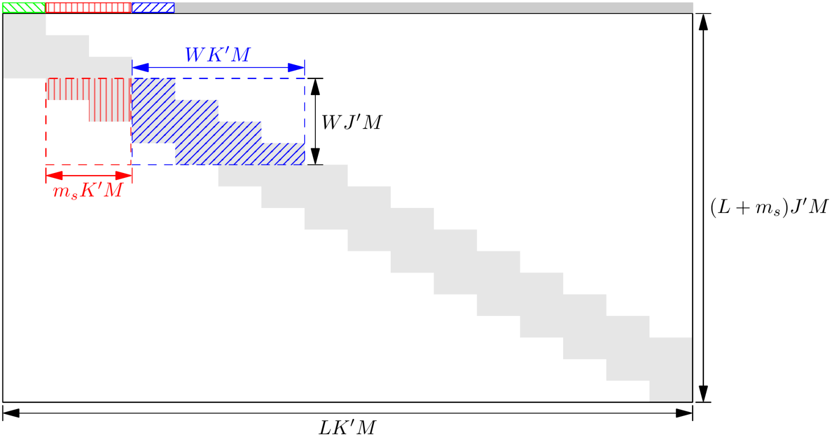


IID Channel (For Reference)

n	Diversity
15000	26
12000	23.9
11000	18.6
10400	24.8
10200	22.3



Windowed Decoding Principle



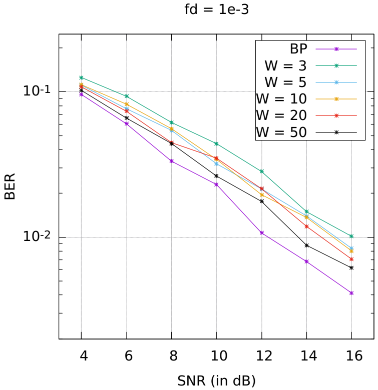
(Aravind R. Iyengar, Marco Papaleo, Paul H. Siegel, et al. "Windowed Decoding of Protograph-Based LDPC Convolutional Codes Over Erasure Channels". In: *IEEE Transactions on Information Theory* 4 (2012))

Comparison With Prior Work

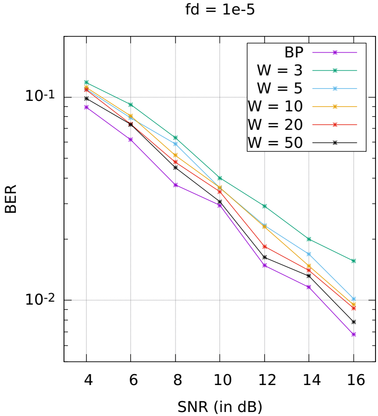
Work	Channel	Variation with W
Aravind R. Iyengar, Marco Papaleo, Paul H. Siegel, et al. "Windowed Decoding of Protograph-Based LDPC Convolutional Codes Over Erasure Channels". In: *IEEE Transactions on Information Theory* 4 (2012), Aravind R. Iyengar, Paul H. Siegel, Rüdiger L. Urbanke, et al. "Windowed Decoding of Spatially Coupled Codes". In: *IEEE Transactions on Information Theory* 4 (2013)	IID binary erasure	yes
Iryna Andriyanova, Najeeb Ul Hassan, Michael Lentmaier, et al. "SC-LDPC codes over the block-fading channel: Robustness to a synchronisation offset". In: 2015	block fading	yes
Sebastian Cammerer, Xiaojie Wang, Yingyan Ma, et al. "Spatially Coupled LDPC Codes and the Multiple Access Channel". In: 2019	IID fading	no
this work	correlated fading	yes

Performance of WD on $\mathcal{C}(3,6,200)$ codes

Fast fading channel

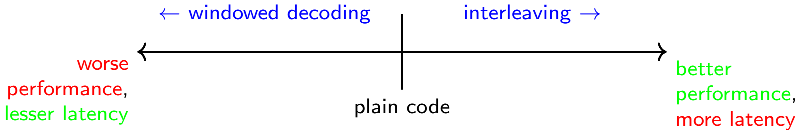


Slow fading channel



BP implementation on MATLAB adapted from NPTEL: LDPC and Polar Codes in 5G Standard (Dr. Andrew Thangaraj), `https://nptel.ac.in/courses/117/106/108106137/`

Summary

1. Performance of SC-LDPC codes $\gg$ LDPC codes
2. 

Thank you